

Together these cover all the four-valuation truth tables – either with just a sentence letter (as in tables 8 and 9) or with a single connective. Here’s a ‘group portrait’, the culmination of our connective spending spree.

$(P \rightarrow P)$	$(P \rightarrow \sim Q)$	$(P \rightarrow Q)$	$(Q \rightarrow P)$	$(\sim P \rightarrow Q)$	$((P \rightarrow \sim Q) \rightarrow \sim(\sim Q \rightarrow P))$	$(P \rightarrow \sim P)$	$(\sim P \rightarrow P)$	$(\sim Q \rightarrow Q)$	$(Q \rightarrow \sim Q)$	$((P \rightarrow Q) \rightarrow \sim(Q \rightarrow P))$	$(P \rightarrow \sim Q)$	$(P \rightarrow Q)$	$(Q \rightarrow P)$	$(\sim Q \rightarrow P)$	$(P \rightarrow P)$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	1	1	1	1	0	1	1	0	0	1	0	0	0	0
1	1	0	1	1	0	0	1	0	1	1	0	1	0	0	0
1	1	1	0	1	0	1	0	1	0	1	0	0	1	0	0
1	1	1	1	0	1	1	0	0	1	0	0	0	0	1	0

And here are all the **expressively adequate, non-redundant languages** built from these connectives.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	1	1	1	1	0	1	1	0	0	1	0	0	0	0
1	1	0	1	1	0	0	1	0	1	1	0	1	0	0	0
1	1	1	0	1	0	1	0	1	0	1	0	0	1	0	0
1	1	1	1	0	1	1	0	0	1	0	0	0	0	1	0

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	$(P \rightarrow P)$	$(P \rightarrow (Q \rightarrow \perp))$	$(P \rightarrow Q)$	$(Q \rightarrow P)$	$((P \rightarrow \perp) \rightarrow Q)$	$((P \rightarrow Q) \rightarrow ((Q \rightarrow P) \rightarrow \perp)) \rightarrow \perp$	$(P \rightarrow \perp)$	$((P \rightarrow \perp) \rightarrow \perp)$	$((Q \rightarrow \perp) \rightarrow \perp)$	$(Q \rightarrow \perp)$	$((P \rightarrow Q) \rightarrow ((Q \rightarrow P) \rightarrow \perp))$	$((P \rightarrow (Q \rightarrow \perp)) \rightarrow \perp)$	$((P \rightarrow Q) \rightarrow \perp)$	$((Q \rightarrow P) \rightarrow \perp)$	$((((Q \rightarrow \perp) \rightarrow P) \rightarrow \perp) \rightarrow \perp)$	$((P \rightarrow P) \rightarrow \perp)$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	0	1	1	1	1	0	1	1	0	0	1	0	0	0	0	
1	1	0	1	1	0	0	1	0	1	1	0	1	0	0	0	
1	1	1	0	1	0	1	0	1	0	1	0	0	1	0	0	
1	1	1	1	0	1	1	0	0	1	0	0	0	0	1	0	

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Equivalent:

$$((P \rightarrow (Q \rightarrow \perp)) \rightarrow \perp) = ((P \rightarrow ((P \rightarrow Q) \rightarrow \perp)) \rightarrow \perp) = (P \wedge Q)$$

$$((P \rightarrow (Q \rightarrow \perp)) \rightarrow (((Q \rightarrow \perp) \rightarrow P) \rightarrow \perp))$$

$$= ((P \rightarrow Q) \rightarrow ((Q \rightarrow P) \rightarrow \perp))$$

$$= (P \leftrightarrow Q)$$

3.16.19:

$$((P \vee Q) \rightarrow (P \wedge Q)) \approx (P \leftrightarrow Q) \approx$$

$$(((P \rightarrow \perp) \rightarrow Q) \rightarrow ((P \rightarrow (Q \rightarrow \perp)) \rightarrow \perp))$$

Indirect Deduction with $\{\rightarrow, \perp\}$:

To show P, assume $(P \rightarrow \perp)$

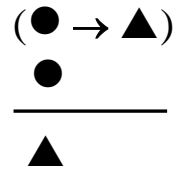
Box closes when: get some sentence, X, and also $(X \rightarrow \perp)$

(NB: from X and $(X \rightarrow \perp)$, get \perp by MP. So just have box close when we get \perp .)

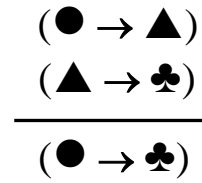
Vel Elim	1. $(P \vee Q)$ 2. $\sim P$ $\therefore Q$	1. $((P \rightarrow \perp) \rightarrow Q)$ 2. $(P \rightarrow \perp)$ $\therefore Q$
Vel Intro	1. P $\therefore (P \vee Q)$	1. P $\therefore ((P \rightarrow \perp) \rightarrow Q)$
Wedge Elim	1. $(P \wedge Q)$ $\therefore P$	1. $((P \rightarrow (Q \rightarrow \perp)) \rightarrow \perp)$ $\therefore P$
Wedge Elim	1. $(P \wedge Q)$ $\therefore Q$	1. $((P \rightarrow (Q \rightarrow \perp)) \rightarrow \perp)$ $\therefore Q$
Wedge Intro	1. P 2. Q $\therefore (P \wedge Q)$	1. P 2. Q $\therefore ((P \rightarrow (Q \rightarrow \perp)) \rightarrow \perp)$
Tilde Elim	1. $\sim\sim P$ $\therefore P$	1. $((P \rightarrow \perp) \rightarrow \perp)$ $\therefore P$
Tilde Intro	1. P $\therefore \sim\sim P$	1. P $\therefore ((P \rightarrow \perp) \rightarrow \perp)$
Modus Tollens	1. $(P \rightarrow Q)$ 2. $\sim Q$ $\therefore \sim P$	1. $(P \rightarrow Q)$ 2. $(Q \rightarrow \perp)$ $\therefore (P \rightarrow \perp)$
1.	1.	1.

Rules of Inference:

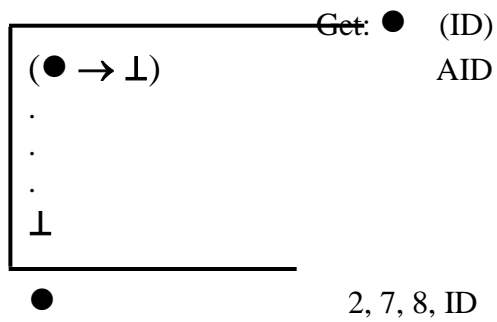
Modus Ponens (MP)



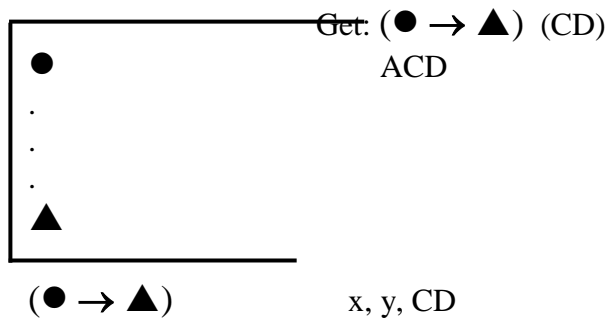
Transitivity (T)



Indirect Deduction



Conditional Deduction



Can get T from CD, MP